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On the coupling $g_{f_0 K^+ K^-}$ and the structure of $f_0(980)$

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Abstract. We use light-cone QCD sum rules to evaluate the strong coupling $g_{f_0 K^+ K^-}$ which enters in several analyses concerning the scalar $f_0(980)$ meson. The result is $6.2 \leq g_{f_0 K^+ K^-} \leq 7.8$ GeV.

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1 Introduction

The nature of light scalar mesons still needs to be unambiguously established [1,2]. Their identification is made problematic since both quark-antiquark ($q\bar{q}$) and non $q\bar{q}$ scalar states are expected to exist in the energy regime below 2 GeV. For example, lattice QCD and QCD sum rule analyses indicate that the lowest lying glueball is a 0^{++} state with mass in the range 1.5-1.7 GeV [3]. Actually, the observed light scalar states are too numerous to be accomodated in a single $q\bar{q}$ multiplet, and therefore it has been suggested that some of them escape the quark model interpretation. Besides glueballs, other interpretations include multiquark states and quark-gluon admixtures.

Particularly debated is the nature of $f_0(980)$. Among the oldest suggestions, there is the proposal that confinement could be explained by the existence of a state with vacuum quantum numbers and mass close to the proton mass [4]. On the other hand, following the quark model and considering the strong coupling to kaons, $f_0(980)$ could be interpreted as an $s\bar{s}$ state [5,6,7,8]. However, this does not explain the mass degeneracy between $f_0(980)$ and $a_0(980)$ interpreted as a $(u\bar{u} - d\bar{d})/\sqrt{2}$ state. A four quark $qq\overline{qq}$ state interpretation has also been proposed [9]. In this case, $f_0(980)$ could either be nucleon-like [10], *i.e.* a bound state of quarks with symbolic quark structure $f_0 = s\bar{s}(u\bar{u} + d\bar{d})/\sqrt{2}$, the $a_0(980)$ being $a_0 = s\bar{s}(u\bar{u} - d\bar{d})/\sqrt{2}$, or deuteron-like, *i.e.* a bound state of hadrons. If f_0 is a bound state of hadrons, it is usually referred to as a $K\overline{K}$ molecule [11,12,13,14]. In the former of these two possibilities mesons are treated as point-like, while in the latter they should be viewed as extended objects. The identification of the f_0 and of the other lightest scalar mesons with the Higgs nonet of a hidden U(3) symmetry has also been suggested [15]. Finally, a different interpretation consists in considering $f_0(980)$ as the result of a process in which strong interaction enriches a pure $\bar{q}q$ state with other components, such as $|K\bar{K}\rangle$, a process known as hadronic dressing [6,16]; such an interpretation is supported in [2,5,6,8,17,18,19].

The radiative $\phi \to f_0\gamma$ decay mode has been identified as an effective tool to discriminate among the various scenarios [10,12,20]. As a matter of fact, if f_0 has a pure strangeness component $f_0 = s\bar{s}$, the dominant $\phi \to f_0\gamma$ decay mechanism is the direct transition, while in the four-quark scenario $\phi \to f_0\gamma$ is expected to proceed through kaon loops with a branching fraction depending on the specific bound state structure [12,20].

An important hadronic parameter is the strong coupling $g_{f_0 K^+ K^-}$. Indeed, the kaon loop diagrams contributing to $\phi \to f_0\gamma$ are expressed in terms of $g_{f_0 K^+ K^-}$, as well as in terms of $g_{\phi K^+ K^-}$ which can be inferred from experimental data on ϕ meson decays. In the present paper, we report on a study [21] devoted to determining $g_{f_0 K^+ K^-}$ by light-cone QCD sum rules [22,23]. Such an analysis is presented in Sect. 2, while comparison with experimental and theoretical determinations is given in Sect. 3.

2 $g_{f_0 K^+ K^-}$ by light cone QCD sum rules

In order to evaluate the strong coupling $g_{f_0 K^+ K^-}$, defined by the matrix element:

$$< K^+(q) K^-(p) | f_0(p+q) >= g_{f_0 K^+ K^-} \quad , \qquad (1)$$

we consider the correlation function

$$T_\mu(p,q) = i \int d^4x\, e^{ip\cdot x} \langle K^+(q) | T[J^K_\mu(x) J_{f_0}(0)] | 0 \rangle \quad , \qquad (2)$$

where $J^K_\mu = \bar{u}\gamma_\mu\gamma_5 s$ and $J_{f_0} = \bar{s}s$. The external kaon state has four momentum q, with $q^2 = M_K^2$. The choice of the $J_{f_0} = \bar{s}s$ current does not imply that $f_0(980)$ has a pure $\bar{s}s$ structure, but it simply amounts to assume that J_{f_0} has a non-vanishing matrix element between the vacuum and f_0 [19,24]. Such a matrix element, as mentioned below, has been derived by the same sum rule method.

Exploiting Lorentz invariance, T_μ can be written in terms of two independent invariant functions, T_1 and T_2: $T_\mu(p,q) = i\,T_1(p^2,(p+q)^2)\,p_\mu + T_2(p^2,(p+q)^2)\,q_\mu$. The general strategy of QCD sum rules consists in representing T_μ in terms of the contributions of hadrons (one-particle states and the continuum) having non-vanishing matrix elements with the vacuum and the currents (J^K_μ and J_{f_0} in the present case), and matching such a representation with a QCD expression computed in a suitable region of the external momenta p and $p+q$ [25].

Let us consider, in particular, the invariant function T_1 that can be represented by a dispersive formula in the two variables p^2 and $(p+q)^2$:

$$T_1(p^2,(p+q)^2) = \int ds ds' \frac{\rho^{had}(s,s')}{(s-p^2)[s'-(p+q)^2]} \,. \quad (3)$$

The hadronic spectral density ρ^{had} gets contribution from the single-particle states K and f_0, for which we define current-particle matrix elements:

$$\langle f_0(p+q)|J_{f_0}|0\rangle = M_{f_0}\tilde{f} \;\; , \;\; \langle 0|J^K_\mu|K(p)\rangle = i f_K p_\mu \;\; , \quad (4)$$

as well as from higher resonances and a continuum of states that we assume to contribute in a domain D of the s, s' plane, starting from two thresholds s_0 and s'_0. Therefore, neglecting the f_0 width, the spectral function ρ^{had} can be modeled as:

$$\rho^{had}(s,s') = f_K M_{f_0}\tilde{f} g_{f_0K^+K^-}\delta(s-M_K^2)\delta(s'-M_{f_0}^2) + \rho^{cont}(s,s')\theta(s-s_0)\theta(s'-s'_0)\,, \quad (5)$$

where ρ^{cont} includes the contribution of the higher resonances and of the hadronic continuum. The resulting expression for T_1 is:

$$T_1(p^2,(p+q)^2) = \frac{f_K M_{f_0}\tilde{f} g_{f_0K^+K^-}}{(M_K^2-p^2)(M_{f_0}^2-(p+q)^2)} + \int_D ds ds' \frac{\rho^{cont}(s,s')}{(s-p^2)[s'-(p+q)^2]} \,. \quad (6)$$

We do not consider possible subtraction terms in (3) as they will be removed by a Borel transformation.

For space-like and large external momenta (large $-p^2$, $-(p+q)^2$) T_1 can be computed in QCD as an expansion near the light-cone $x^2=0$. The expansion involves matrix elements of non-local quark-gluon operators, defined in terms of kaon distribution amplitudes of increasing twist. [1] The first few terms in the expansion are retained, since the higher twist contributions are suppressed by powers of $1/(-p^2)$ or $1/(-(p+q)^2)$. For the resulting expression for T_1, obtained to twist four accuracy, we refer to [21].

The sum rule for $g_{f_0K^+K^-}$ follows from the approximate equality of (6) and the computation of T_1 in QCD. Invoking global quark-hadron duality, the contribution of the continuum in (6) can be identified with the QCD contribution above the thresholds s_0, s'_0. This allows us to isolate the pole contribution in which the coupling appears. Such a matching is improved performing two independent Borel transformations with respect to the variables $-p^2$ and $-(p+q)^2$, with M_1^2, M_2^2 the Borel parameters associated to the channels p^2 and $(p+q)^2$, respectively. In order to identify the continuum contribution with the QCD term, a prescription has been proposed in [27], consisting in considering the symmetric values $M_1^2 = M_2^2 = 2M^2$. Such a prescription is not adeguate in our case, where the Borel parameters correspond to channels with different mass scales and should not be constrained to be equal. A different method has been suggested in [21] for the present calculation, exploiting the property of the leading twist wave functions of being polynomials in u (or $1-u$). The subleading twist terms represent a small contribution to the QCD side of the sum rule, and hence the calculation can leave them unaffected.

The main nonperturbative input quantities in the final sum rule are the kaon light-cone wave functions. A theoretical framework for their determination relies on an expansion in terms of matrix elements of conformal operators [28]. For the kaon we took into account the meson mass corrections, related to the parameter $\rho^2 = \frac{m_s^2}{M_K^2}$, worked out in [29]. For details about the distribution amplitudes we refer to the [27,29]. In the analysis of the sum rule we use $m_s(1\,\text{GeV}) = 0.14$ GeV [30], $M_K = 0.4937$ GeV, $M_{f_0} = 0.980$ GeV, $f_K = 0.160$ GeV and $\tilde{f} = (0.180 \pm 0.015)$ GeV [19]. The threshold parameter s_0 is varied around the value $s_0 = 1.1$ GeV^2 fixed from the determination of f_K using two-point sum rules [31]. The final sum rule provides $g_{f_0K^+K^-}$ as a function of the Borel parameters M_1^2, M_2^2. A stability region where the outcome does not depend on M_i^2 can be selected. Such a region does not correspond to the line $M_1^2 = M_2^2$, but to the range $0.8 \le M_1^2 \le 1.6$ GeV^2 with M_2^2 extending up to $M_2^2 \simeq 5$ GeV^2. Varying M_1^2 and M_2^2 in this region, and changing the values of the thresholds and of the other parameters, we obtain the result depicted in fig.1, which can be quoted as $6.2 \le g_{f_0K^+K^-} \le 7.8$ GeV.

Let us briefly discuss the uncertainties affecting the numerical result. We neglected the $SU(3)_F$ breaking effects which render the kaon distribution amplitudes asymmetric with respect to the middle point; such a neglect should have a minor role in our approach, as discussed in [21]. Another uncertainty is related to the value of the strange quark mass, m_s; since the dependence of the sum rule on m_s mainly involves the ratio M_K^2/m_s, one can fix this ratio using chiral perturbation theory, obtaining results in the same range quoted for $g_{f_0K^+K^-}$.

3 Comparison with other results and conclusions

The various determinations of $g_{f_0K^+K^-}$ form a very complex scenario. A collection of experimental results is pro-

[1] The short-distance expansion of the 3-point function of one scalar $\bar{s}s$ and two pseudoscalar $\bar{s}i\gamma_5 q$ densities was considered in [26]. The present calculation mainly differs for the possibility of incorporating an infinite series of local operators [23].

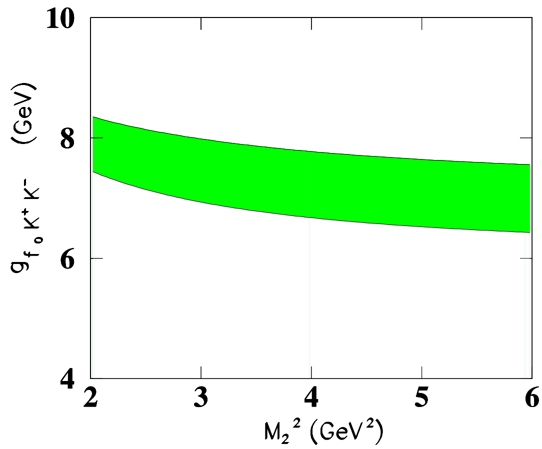

Fig. 1. $g_{f_0K^+K^-}$ as a function of the Borel parameter M_2^2, varying: $1.05 \leq s_0 \leq 1.15$ GeV^2 and $0.7 \leq M_1^2 \leq 2.0$ GeV^2

vided in Table 1. In the case of KLOE Collaboration, two results are reported, corresponding to two different fits performed in the analysis of the data, indicated with (A) and (B). The difference mainly consists in the inclusion of the σ contribution in fit (B). Such a result is the one affected by the smallest uncertainty, and seems to point towards large values of $g_{f_0K^+K^-}$. Theoretical results also lie in a rather large range of values, from 2 GeV up to 7 GeV. For a detailed discussion we refer to [21], while an analysis based on experimental data can be found in [37]. The outcome of light-cone QCD sum rule analysis, reported here, is in keeping with a large value for the coupling. The uncertainty affecting the result is intrinsic of the method and does not allow a better comparison with data. However, the analysis confirms a peculiar aspect of the scalar states, i.e. their large hadronic couplings, thus pointing towards a scenario in which the process of hadronic dressing is favoured. However, since the most accurate experimental data stem from the investigation of $\phi \to f_0\gamma$, it is mandatory to wait for the study of unrelated processes, namely the combined analysis of D_s decays to pions and kaons, which could be performed, for example, at the B-factories.

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Table 1. Experimental determinations of $g_{f_0K^+K^-}$ using different physical processes

Collaboration	process	$g_{f_0K^+K^-}$ (GeV)	Ref.
KLOE	$\phi \to f_0\gamma\,(A)$	$4.0 \pm 0.2\,(A)$	[32]
	$\phi \to f_0\gamma\,(B)$	$5.9 \pm 0.1\,(B)$	
CMD-2	$\phi \to f_0\gamma$	4.3 ± 0.5	[33]
SND	$\phi \to f_0\gamma$	5.6 ± 0.8	[34]
WA102	pp	2.2 ± 0.2	[35]
E791	$D_s \to 3\pi$	0.5 ± 0.6	[36]

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